Multivariable Calculus

Quiz 1 **SOLUTIONS**

Consider $\triangle PQR$ with points $P = (1, -2, 1)$, $Q = (2, 1, 3)$, and $R = (1, 0, 4)$.

1) Find the measure of angle $\angle PRQ$. You can express your answer as the inverse cosine of some value.

Solution: We need the following vectors:

$$\overrightarrow{PR} = \langle 0, -2, -3 \rangle \text{ and } \overrightarrow{QR} = \langle 1, 1, -1 \rangle.$$

Note that $\|\overrightarrow{PR}\| = \sqrt{13}$, $\|\overrightarrow{QR}\| = \sqrt{3}$, and $\overrightarrow{PR} \cdot \overrightarrow{QR} = 1$. Since we know that

$$\overrightarrow{PR} \cdot \overrightarrow{QR} = \|\overrightarrow{PR}\| \|\overrightarrow{QR}\| \cos(\theta),$$

we know that

$$\theta = \arccos\left(\frac{\overrightarrow{PR} \cdot \overrightarrow{QR}}{\|\overrightarrow{PR}\| \|\overrightarrow{QR}\|}\right) = \arccos\left(\frac{1}{\sqrt{39}}\right) \approx 80.79°$$

2) Find the area of $\triangle PQR$.

Solution:

$$\begin{aligned}
\text{Area}(\triangle PQR) &= \frac{1}{2}\|\overrightarrow{PR} \times \overrightarrow{QR}\| \\
&= \frac{1}{2}\left\| \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -2 & -3 \\ 1 & 1 & -1 \end{vmatrix} \right\| \\
&= \frac{1}{2}\left\| 5\hat{\imath} - 3\hat{\jmath} + 2\hat{k} \right\| \\
&= \frac{1}{2}\sqrt{25 + 9 + 4} = \frac{\sqrt{38}}{2}
\end{aligned}$$

3) Give the equation of the plane that contains $\triangle PQR$.
HINT: Hopefully your work in problem 2 is helpful here!

Solution: From the previous problem, we can take $\vec{n} = \langle 5, -3, 2 \rangle$ as the normal vector for the plane. Using point P as the given point (any choice would do) we have

$$\begin{aligned}
\langle 5, -3, 2 \rangle \cdot \langle x - 1, y + 2, z - 1 \rangle &= 0 \\
5(x - 1) - 3(y + 2) + 2(z - 1) &= 0 \\
5x - 3y + 2z &= 13.
\end{aligned}$$